and from a matrix exponential time-marching structural dynamics solver to evaluate the dynamic response of the system to these aerodynamic loads. The trained neural networks could function at a varying torsional stiffness within a specified range of stiffness variation. This array of neural networks was utilized to detect flutter in the aeroelastic system and suppress flutter by using the concept of a time-varying torsional stiffness.

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Intergrid Transformation for Aircraft Aeroelastic Simulations

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Introduction

COMPUTATIONAL aeroelasticity requires the coupling of structural and fluid codes. Usually the structural grid and the fluid surface grids do not coincide, and hence a data exchange method is required that interfaces the loads and deformation information between the two grids. A review of some of these transformation methods is available in Hounjet and Meijer,[1] Guruswamy,[2] Smith et al.,[3] and Potsdam and Guruswamy.[4] The simulation of the aeroelastic response of wing-alone cases has become common.

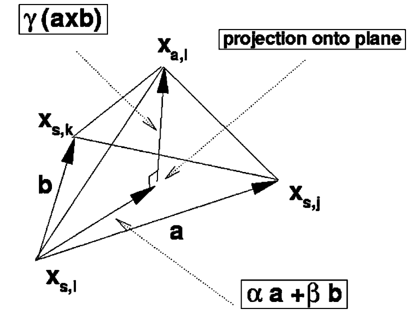

Fig. 1 Constant-volume-tetrahedron transformation.

Simulation in the transonic flow regime requires nonlinear aerodynamic codes, which in turn are sensitive to changes in external geometry. For a complete aircraft, multicomponents need to be transformed without introducing holes in the aerodynamic surface. A reliable transformation method for aircraft geometries is proposed in this Note and is demonstrated for the structural dynamics model of Huang and Zan,[5] a generic model for the F-16 aircraft.

Constant-Volume Tetrahedron

The constant-volume-tetrahedron (CVT) scheme is a transformation technique proposed in Goura[6] and Goura et al.[7] A surface element consisting of the three nearest structural grid points $\boldsymbol{x}_{s,i}(t)$, $\boldsymbol{x}_{s,j}(t)$, and $\boldsymbol{x}_{s,k}(t)$ to a given fluid grid point $\boldsymbol{x}_{a,l}(t)$ is identified. Once the structural grid points are identified and associated with the fluid grid point, the position of $\boldsymbol{x}_{a,l}$ is given by the expression

$$\boldsymbol{x}_{a,l} - \boldsymbol{x}_{s,i}(t) = \alpha\boldsymbol{a} + \beta\boldsymbol{b} + \gamma\boldsymbol{d} \tag{1}$$

where $\boldsymbol{a} = \boldsymbol{x}_{s,j} - \boldsymbol{x}_{s,i}$, $\boldsymbol{b} = \boldsymbol{x}_{s,k} - \boldsymbol{x}_{s,i}$, and $\boldsymbol{d} = \boldsymbol{a} \times \boldsymbol{b}$. Here the term $\alpha\boldsymbol{a} + \beta\boldsymbol{b}$ represents the location of the projection of $\boldsymbol{x}_{a,l}$ onto the structural triangle and $\gamma\boldsymbol{d}$ is the component out of the plane of this triangle, as shown in Fig. 1. In the preceding the values of α, β, and γ are calculated as

$$\alpha = \frac{|\boldsymbol{b}|^2(\boldsymbol{a}\cdot\boldsymbol{c}) - (\boldsymbol{a}\cdot\boldsymbol{b})(\boldsymbol{b}\cdot\boldsymbol{c})}{|\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\cdot\boldsymbol{b})(\boldsymbol{a}\cdot\boldsymbol{b})} \tag{2}$$

$$\beta = \frac{|\boldsymbol{a}|^2(\boldsymbol{b}\cdot\boldsymbol{c}) - (\boldsymbol{a}\cdot\boldsymbol{b})(\boldsymbol{a}\cdot\boldsymbol{c})}{|\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\cdot\boldsymbol{b})(\boldsymbol{a}\cdot\boldsymbol{b})} \tag{3}$$

$$\gamma = \frac{(\boldsymbol{c}\cdot\boldsymbol{d})}{|\boldsymbol{d}|^2} \tag{4}$$

As the structure deforms, the values of the structural grid points change. The location of the fluid grid point is recalculated using Eq. (1) with the following assumptions. First, the projection of the fluid point is forced to move linearly in the structural triangle by fixing α and β at their initial values. The value of γ, which scales the out-of-plane component, is calculated to ensure that the volume of the tetrahedron formed by the three structural and one fluid points remains constant. If the fluid and the structural points are planar, then the expression reduces to linear interpolation for the position of the fluid point.

This method has previously been tested for the aeroelastic response of isolated wings in Goura et al.[8] The extension to complete aircraft configurations is considered in the following section.

Transformation for Complete Aircraft

A version of the CVT that can do the transformation for a complete aircraft with the minimum of manual intervention and that preserves the surface mesh, particularly at junctions between components, is required. The insight for the method is provided by the paper of Melville,[9] which treats the aircraft components in a hierarchy.

The first stage of the method is to partition the fluid and structural points into levels associated with components. The primary component is the fuselage because all of the other parts of the aircraft

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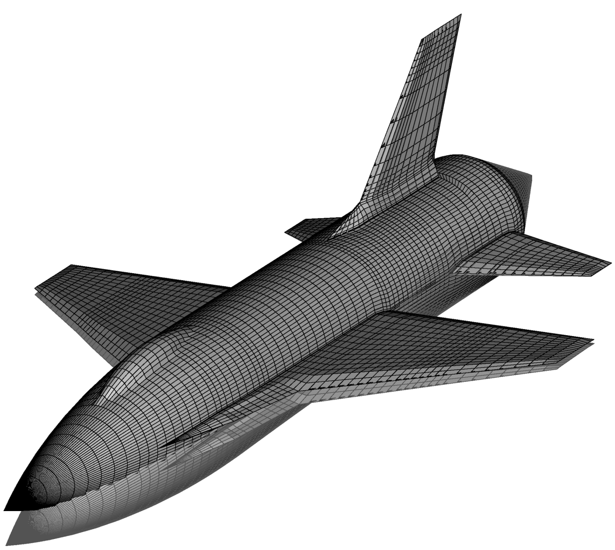
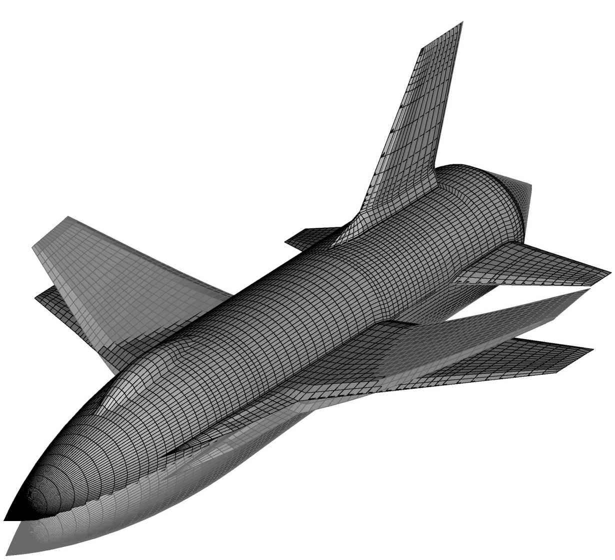
are connected to it. The fluid and structural grid points on the fuselage are therefore designated as being of level 1. Next, the wings, tail planes, and fin are connected to the fuselage, and the fluid and structural grid points on these components and the fuselage are designated level 2. The idea of the hierarchy is that level 2 points have a primary motion because they are connected to the fuselage and a secondary motion caused by their own elasticity. Extra components attached to the wing, such as fuel tanks and stores, would be designated level 3, with their primary motion possibly being caused by the fact that they are attached to the wing, although only two-level transformations are considered in this Note. At this stage a number of subsets of points have been defined for the fluid and structural grids, with one subset for each level.

The first stage for the CVT is to associate each fluid point with three structural points. This is done by defining a triangulation of the structural grid and then searching for the nearest centroid to each aerodynamic point. This search can be restricted to the structural triangles in each level, defining level 1 and 2 mappings. In the current case the level one mapping drives all points in the fluid grid using structural points on the fuselage only. The level 2 mapping is equivalent to the original CVT method applied to all fluid grid points without any restriction on the choice of the driving structural points. The transformation of a wing bending mode is shown in Fig. 2 using successively the first- and second-level mappings. The first-level

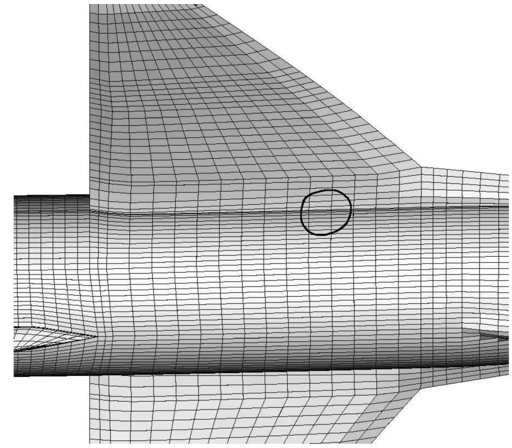


a) Level 1

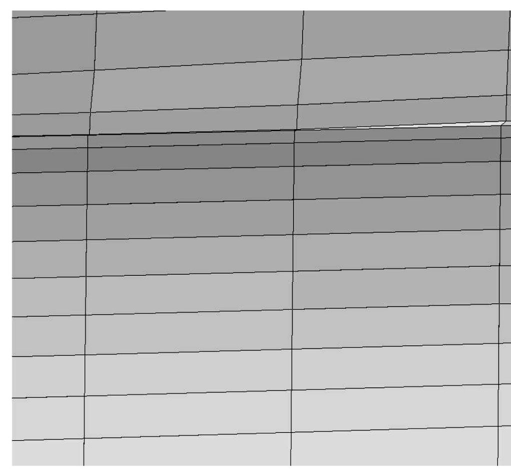


b) Two step

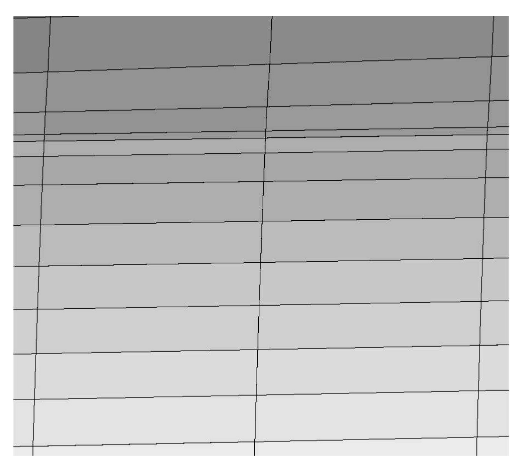
Fig. 2 Transformation.



a) Circle indicates area of interest



b) One-level transformation

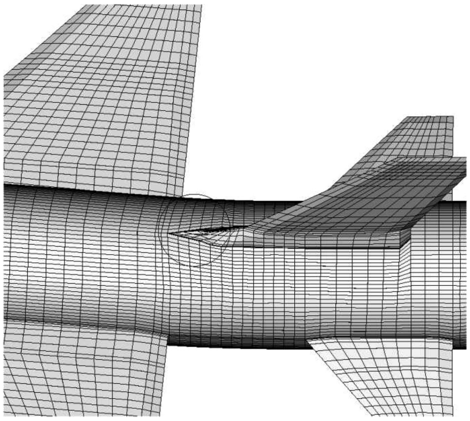


c) Fuselage wing interface close-up

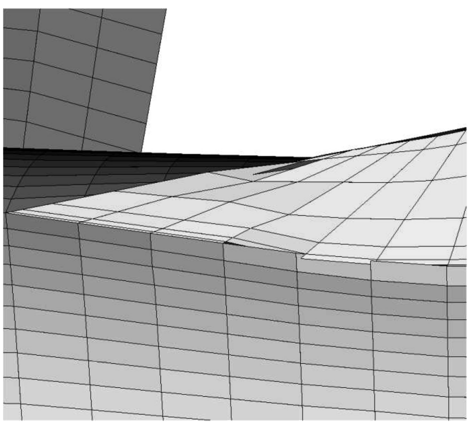
Fig. 3 Fuselage wing interface close-up views.

mapping leads to the fluid grid motion following the fuselage, with the wings being moved in a rigid fashion. The second-level mapping introduces the wing bending as well, with the motion of the fuselage the same as that arising from the first-level mapping.

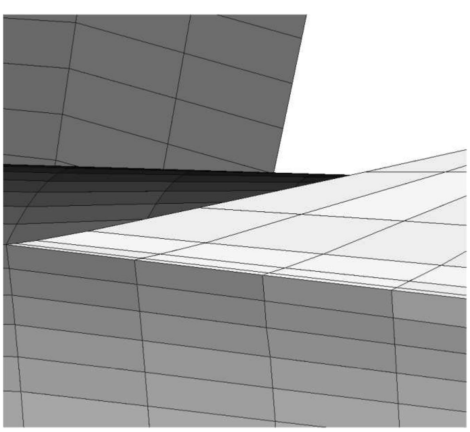
A problem with the level 2 mapping arises at junctions between components. This is illustrated in Fig. 3b. A second problem arises where the fin is attached to the fuselage, as shown in Fig. 4b. For the level 2 mapping the nodes off the fuselage are being driven by different components in the structural model from those actually on the junction, which are driven by the fuselage. This leads to a small but disastrous distortion of the grid in the junction regions arising from transforming the points from different structural components. Using



a) Circle indicates area of interest



b) One-level transformation



c) Two-level blended transformation

Fig. 4 Fuselage vertical fin interface close-up views.

the level 1 mapping treats all points in a consistent way and maintains the grid quality in the junction regions as a result. However, the level 1 mapping misses all effects introduced by the elasticity of the nonfuselage components because these structural components are not used to drive the fluid surface grid. A new method is therefore needed to correctly transform the complete deformation whilst avoiding the problems at junctions.

The basis for the method is the observation that the level 1 and 2 transformed mode shapes on level 2 components in regions close to the fuselage are almost identical. This follows from the observation of Melville[9] that the fuselage drives the wing motions, and this effect is dominant close to the wing root as opposed to any wing alone elastic effects. The method therefore blends the level 1 and 2 transformed fluid points, giving priority to the level 1 transformation as we approach the fuselage. (In general the level m transformation is given priority as the level m component is approached.) This means that in the junction region the fluid grid is transformed from the fuselage structural model rather than the wing structural model.

Denote the transformed deflection for a fluid point $\boldsymbol{x}_{a,l}$ using the mth level mapping as $\delta \boldsymbol{x}^m_{a,l}$. The blending used to give the final transformed displacement is given as

$$\delta \boldsymbol{x}_{a,l} = \sum_{m=1}^{n} w_{m,l} \delta \boldsymbol{x}^m_a \tag{5}$$

where the weights for the blending $w_{m,l}$ must add to one. To define the values of the weights for level m, we need to consider the distance from the components associated with that level. Define the nearest distance of the point $\boldsymbol{x}_{a,l}$ to all of the points in level m by $d_{m,l}$. It is a simple matter to calculate $d_{m,l}$ by searching over the fluid points defined in level m for the nearest point. If $\boldsymbol{x}_{a,l}$ actually belongs to level m, then $d_{m,l} = 0$. Then, the weights for blending the two levels of transformation in the current test case are computed from

$$w_{1,l} = e^{-10 d_{m,l}} \tag{6}$$

$$w_{2,l} = 1 - w_{1,l} \tag{7}$$

For points on the fuselage, the entire weight will be put on the fuselage-driven transformation; for points close to the fuselage, most weight will be given to the fuselage-driven transformation, and otherwise most weight is given to the level 2 component-driven transformation. Looking to the junction region, the blended transformation has avoided the folded grid as required (see Fig. 3c). Also, the fin now remains cleanly attached to the fuselage in contrast with the level two transformation (Fig. 4c). Because the cost of computing the original CVT transformation is small, the cost of applying the new multilevel scheme is also small. On cost grounds there is an objection to using the exponential function in the weighting, but the weights are calculated as part of a preprocessing step so this is insignificant.

Conclusions

This Note has considered the generalization of a transformation method for passing information between structural and aerodynamic surface grids for aeroelastic simulation. The initial method was developed for isolated aircraft components. This has been extended to consider multiple connected components by allowing different parts of the structural model to drive the fluid surfaces in a hierarchical manner, and then blending the resulting fluid positions to ensure that the different parts of the fluid surface mesh remain attached. The method has been applied for wings, tailplanes, and a vertical fin attached to a fuselage. Future work will consider components attached to the wing (e.g., fuel tanks or stores).

The next stage of this work is to apply the transformation to the time-domain aeroelastic simulation of complete aircraft.

Acknowledgments

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Characteristics of Sensitivity Analysis of Repeated Frequencies

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I. Introduction

THE design sensitivity analysis of a natural frequency of a vibration system has become increasingly important for various gradient-based optimization algorithms. For instance, it is used for studying the effect of a design modification, finding the search direction, and constructing the frequency approximation. However, the occurrence of repeated frequencies constitutes one of the main difficulties in structural optimizations with dynamic frequency constraints because of nondifferentiability of a repeated frequency.[1] When repeated frequencies arise in a system, the related vibration mode shape can not be uniquely determined. Any linear combination of the modes is still valid for the repeated frequency. The present paper investigates the design sensitivity computation of a repeated frequency in the vibration mode space.

It is widely recognized that a repeated frequency is not generally differentiable in the common sense, that is, the Frechet derivative does not exist. Only directional derivatives can be obtained.[1] The nondifferentiability of a repeated frequency may be attributed to the fact that the mode corresponding to a repeated frequency has a great deal of uncertainty compared to a distinct one. Nonetheless, repeated frequencies are often involved in dynamic systems. They are associated with the structural symmetry or induced by the frequency evolution in the design optimization.[2] Haug et al.[1] and Choi et al.[3] demonstrated that a repeated frequency is directionally differentiable. Mills-Curran[4] found that the sensitivities of a repeated frequency can be evaluated by solving a subeigenvalue problem. Seyranian et al.[5] dealt with the frequency sensitivity by means of a design perturbation approach. Friswell[6] demonstrated that the first-order Taylor series of a repeated frequency is less valid for several design parameters. Sergeyev and Mroz[2] computed the design sensitivity along a special path in the design space. Pedersen and Nielsen[7] made an attempt to describe the sensitivity computation in the eigenspace. To avoid mode selection, Prells and Friswell[8] even presented an algorithm to calculate the sensitivity without explicit use of vibration modes.

The objective of this Note is to reveal and demonstrate some interesting properties for the design sensitivity of a repeated frequency. It will be shown that the sensitivity calculation can be obtained with additional physical interpretations in the vibration mode space. Furthermore, the characteristics of the sensitivity analysis will be investigated systematically. These characteristics will lay the foundations for the standard gradient-based optimization algorithms. In accordance with these characteristics, the maximum of the fundamental frequency can be simply determined. By analogy, the proposed approach can be extended straightforwardly to the sensitivity analysis of repeated buckling load factors.

II. Sensitivity Analysis of a Natural Frequency

When a dynamic problem is considered, the eigenvalue equation is given as

$$\left([K] - \omega_j^2[M]\right)\{\phi\}_j = \{0\}, \qquad j = 1, \ldots, n \tag{1}$$

where $[K]$ and $[M]$ are the global stiffness and mass matrices, respectively, which are assumed real, symmetric, and differentiable; ω_j is the jth natural frequency, $\{\phi\}_j$ is the corresponding vibration mode, and n is the number of degrees of freedom. The frequencies are real and ordered in increasing magnitude:

$$0 \le \omega_1^2 \le \omega_2^2 \le \cdots \le \omega_n^2 \tag{2}$$

All of the modes are $[M]$ orthonormalized so that

$$\{\phi\}_i^T[K]\{\phi\}_j = \omega_j^2\delta_{ij}$$
$$\{\phi\}_i^T[M]\{\phi\}_j = \delta_{ij}, \qquad i, j = 1, \ldots, n \tag{3}$$

where δ_{ij} is the Kronecker delta.

A. Distinct Frequency Sensitivity

Suppose we have a distinct eigenpair (ω_j, $\{\phi\}_j$), the first-order derivative of the frequency with respect to a design parameter x is computed as[4]

$$\frac{\partial\omega_j^2}{\partial x} = \{\phi\}_j^T\left(\frac{\partial[K]}{\partial x} - \omega_j^2\frac{\partial[M]}{\partial x}\right)\{\phi\}_j, \qquad j = 1, \ldots, n \tag{4}$$

In most cases, Eq. (4) can be simplified at the element level,

$$\frac{\partial\omega_j^2}{\partial x} = \sum_{e=1}^{n_k}\{\phi_e\}_j^T\left(\frac{\partial[k_e]}{\partial x} - \omega_j^2\frac{\partial[m_e]}{\partial x}\right)\{\phi_e\}_j, \qquad j = 1, \ldots, n \tag{5}$$

where $[k_e]$ and $[m_e]$ are the element stiffness and mass matrices, respectively; $\{\phi_e\}_j$ is the jth mode of the eth element, which contains only the related components of $\{\phi\}_j$; and n_k is the number of elements related to the design parameter. Because of the uniqueness of the mode shape, the design sensitivity can be calculated definitely and explicitly.

B. Repeated Frequency Sensitivity

For simplicity of presentation, we shall only treat a double repeated frequency with the proposed methodology. The derivation for higher-order multiplicity can be performed in a similar way. Assume that a double repeated frequency and its corresponding modes are

$$\left(\omega^2, \{\phi\}_1, \{\phi\}_2\right) \tag{6}$$

No special restrictions are imposed on the mode selections except the $[M]$ orthonormalization in compliance with Eq. (3). Therefore, as is well known, $\{\phi\}_1$ and $\{\phi\}_2$ can span a two-dimensional eigenspace, which is a subspace of the structural mode space. Any

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